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Gauge theory of dislocations and disclinations for planar elastic systems

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Abstract. Making use of the gauge approach we construct the continuum model for the description of topological defects in planar elastic systems. The equations of motion for defect systems in the presence of both dislocation and disclination fields are presented. The exact vortex-like solution for static disclinations is obtained. The Schrödinger equation in the presence of a static disclination vortex is studied. We show that electrons (holes) can acquire a topological phase which depends on the disclination flux. When the interaction between long-wave electron fields and acoustic waves is present, the electrons with $E < E_F$ are found to be localized. The scattered states (at $E > E_F$) acquire an additional phase shift due to the deformation potential.

1. Introduction

At the present time gauge constructions play an essential role in the description of different phenomena in condensed matter physics (see, e.g. [1]). One of the modern trends is the gauge theory of dislocations and disclinations constructed first in a closed form by Edelen and Kadić [2]. This theory enables us to describe the continuum elastic media with continuously distributed topological defects in a self-consistent way. According to [2], the space group G_3 that includes both the translations and rotations in three space dimensions ($G_3 = SO(3) \triangleright T(3)$ for $d = 3$) can be considered as a gauge group. The dislocation fields are associated with the inhomogeneous action of the group T(3) whereas the disclination fields are associated with the inhomogeneous action of the group SO(3). One of the basic concepts of the Edelen–Kadić (EK) gauge theory is the concept of the Yang–Mills minimal coupling theory.

The aim of the present paper is to construct the gauge model for planar elastic systems with dislocations and disclinations continuously distributed in materials on the basis of a direct analogy with the (3+1)-dimensional EK model. The gauge group takes the following form in two space dimensions: $G_2 = SO(2) \triangleright T(2)$. It should be noted that the group SO(2) is Abelian instead of the non-Abelian group SO(3) for $d = 3$. Two-dimensional problems play an important role in condensed matter physics. Planar systems with topological defects are of interest in liquid crystals, polymers, layered crystals, etc. In recent years, the defect-mediated melting in two dimensions has been intensively studied (see, e.g. [3]). The lattice gauge model describing this phenomenon was proposed first in [4] and studied in detail in [5, 6]. Besides, the (2+1)-dimensional systems of relativistic field theory and condensed matter physics are intensively studied due to topologically non-trivial effects which can exist in two space dimensions (see, e.g. [7]).

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As was proposed in our previous paper [8], there may exist an attractive possibility for the solid state realization of the Aharonov–Bohm (AB) effect [9] in planar elastic systems with disclinations. Namely, we have shown that electrons (or holes) in planar elastic systems can acquire a topological phase if the disclination vortices are present. The reason is that the topology of space changes in the presence of dislocations and/or disclinations. As has been shown first in [10, 11], the topologically singular character of dislocated crystals causes a new type of scattering process of electrons (phase-dismatching effect). Moreover, it has been shown recently [12] that the effects of Berry's geometrical phase [13] can be observed in high-energy electron diffraction in a deformed crystal lattice with a screw dislocation.

The plan of the paper is as follows. In section 2 the field equations for defect dynamics in two space dimensions are constructed. In section 3 we study rotational defects and an exact solution for static disclination vortices is found. The electronic properties of planar elastic systems with defects are studied in section 4. For this purpose we use the general approach developed in [14] where the electronic fields were introduced in a gauge invariant form. We analyse the Schrödinger equation in an external field due to a static disclination vortex. We show that the wavefunction of an electron interacting with a disclination flux acquires a phase change like that in the AB effect. Note that in many respects there is a close analogy of this phenomenon with the gravitational AB effect intensively studied at present (see, e.g. [15–17]). The role of the interaction between long-wave electronic fields and acoustic waves is studied in section 5.

2. The gauge model

We will start with the continuum Lagrangian of the elasticity theory that is invariant under the inhomogeneous action of the gauge group G_2. Note that it has the same form as in (3+1) dimensions and can be written in the isotropic case as (see, for detail, [2]):

$$L = L_\chi + L_\phi + L_W \tag{1}$$

where

$$L_\chi = (\rho_0/2) B^i_3 \delta_{ij} B^j_3 - [\lambda (E_{AB}\delta^{AB})^2 + 2\mu E_{AB}\delta^{AC}\delta^{BD}E_{CD}]/8 \tag{2}$$

describes the elastic properties of the material,

$$L_\phi = -(s_1/2)\delta_{ij} D^i_{ab} k^{ac} k^{bd} D^j_{cd} \tag{3}$$

describes the dislocations, and

$$L_W = -(s_2/2) F_{ab} g^{ac} g^{bd} F_{cd} \tag{4}$$

describes the disclinations. The strain tensor in (2) is determined to be

$$E_{AB} = B^i_A \delta_{ij} B^j_B - \delta_{AB} \tag{5}$$

where

$$B^i_a = \partial_a \chi^i + \varepsilon^i_j \chi^j W_a + \phi^i_a \tag{6}$$

is the distortion tensor. In (6) $\partial_a \chi^i$ describes the integrable part of the distortion, the second term arises from the inhomogeneous action of the rotation group SO(2), and the third arises from the breaking of homogeneity of the action of the translation group T(2). The state vector $\chi^i(X^a)=\chi^i(X^A, T)$ in (6) characterizes the configuration at time T in terms of the coordinate cover (X^A) of a reference configuration, W_a are the compensating gauge fields associated with disclination fields, whereas ϕ^i_a are associated with dislocation fields. We have used here the same notation as in [2] adapted to $d=2$. The summation over repeated indices is assumed.

Tensors D^i_{ab} and F_{ab} are determined as follows:

$$D^i_{ab}=\partial_a\phi^i_b-\partial_b\phi^i_a+\varepsilon^i_j(W_a\phi^j_b-W_b\phi^j_a+F_{ab}\chi^j) \tag{7}$$

and

$$F_{ab}=\partial_a W_b-\partial_b W_a. \tag{8}$$

In (2)-(4) λ and μ are the Lamé constants, ρ_0 is the mass density in the reference configuration, s_1 and s_2 are the coupling constants, ε^i_j are the generating matrices of the group SO(2): ε^i_j is a completely antisymmetric tensor, $\varepsilon^1_2=1$. In (4) the quantities g^{ab} are given by $g^{AB}=-\delta^{AB}$, $g^{33}=1/\zeta$ and $g^{ab}=0$ for $a\neq b$, whereas in (3) $k^{AB}=-\delta^{AB}$, $k^{33}=1/y$ and $k^{ab}=0$ for $a\neq b$. The parameters ζ and y are two positive 'propagation parameters' [2].

Let us write the Euler–Lagrange equations of defect dynamics. The variation of (1) with respect to χ^i gives

$$\partial_3 p_i-\partial_A\sigma^A_i=\varepsilon^j_i(W_3p_j-W_A\sigma^A_j+F_{ab}R^{ab}_j) \tag{9}$$

where the explicit expression for the stress tensor

$$\sigma^A_i=\tfrac{1}{2}\delta^A_B\delta_{ij}(\partial_C\chi^j+\varepsilon^j_k W_C\chi^k+\phi^j_C)(\lambda\delta^{BC}\delta^{FD}E_{FD}+2\mu\delta^{RB}\delta^{SC}E_{RS}) \tag{10}$$

and the momentum

$$p_i=\rho_0\delta_{ij}(\partial_3\chi^j+\varepsilon^j_k W_3\chi^k+\phi^j_3). \tag{11}$$

In (9), R^{ab}_i is determined as follows:

$$R^{ab}_i=\partial L/\partial D^i_{ab}=-s_1\delta_{ij}k^{ac}k^{bd}[\partial_c\phi^j_d-\partial_d\phi^j_c+\varepsilon^j_k(W_c\phi^k_d-W_d\phi^k_c)+\varepsilon^j_kF_{cd}\chi^k]. \tag{12}$$

Note that (9) are the equations of balance of the linear momentum. When W_a are equal to zero (pure dislocated material), (9) are reduced to the form $\partial_A\sigma^A_i=\partial_3 p_i$ well known in classical elasticity theory.

The Euler–Lagrange equations in ϕ^i_a are

$$\partial_a R^{ab}_j-\varepsilon^i_j W_a R^{ab}_i=Z^b_j/2 \tag{13}$$

where R^{ab}_i is determined above, $Z^a_i=\partial L/\partial B^i_a$, and

$$Z^A_i=-\sigma^A_i \qquad Z^3_i=p_i. \tag{14}$$

The variation with respect to W_a gives

$$\partial_a(G^{ab}+\varepsilon^i_j R^{ac}_i\chi^j)=T^b/2 \tag{15}$$

where $G^{ab}=\partial L/\partial F_{ab}$ and $T^a=(\partial L/\partial W_a)|_{F_{ab}}$.

Additionally to the Euler–Lagrange equations, we write here an important relationship between dislocations, disclinations, and stresses

$$T^a=\varepsilon^i_j(Z^a_i\chi^j+2R^{ab}_i\phi^j_b) \tag{16}$$

and the integrability conditions

$$\varepsilon^i_j\sigma^A_i B^j_A=0 \tag{17}$$

which determine the balance of the moment of momentum. As was shown in [2], two types of the boundary conditions may be written for field equations: (a) the Dirichlet data (traction-free spatial boundaries), and (b) the homogeneous Neumann data (zero initial and final momentum). Let us note also that the theory [2] has been constructed so that non-exact gauge conditions must be satisfied. Namely,

$$X^a \phi_a^i = 0 \qquad X^a W_a = 0. \tag{18}$$

Finally, we have constructed in this section the field theory for defects in planar elastic systems. It is clear that field equations given above are a system of coupled nonlinear differential equations which is very difficult to solve in the general case. However, in three space dimensions we have found an exact monopole-like solution for static disclinations [18]. Since a vortex is in many ways a planar analogue of the monopole, we hope to find an exact vortex solution in a planar case.

3. Rotational defects (static case)

Rotational defects are known to play an important role in defect materials (see, e.g. [19]). Let us consider the disclination Lagrangian $L = L_\chi + L_W$ where the dislocation fields ϕ_a^i are to be ignored from the beginning. In this case, the Euler-Lagrange equation (13) should be eliminated whereas (9) and (15) can be rewritten as follows:

$$\partial_a G^{ab} = \tfrac{1}{2}\varepsilon_j^i Z_i^b \chi^j \tag{19}$$

and

$$\partial_a Z_i^a = \varepsilon_i^j Z_j^a W_a. \tag{20}$$

Note that this self-consistent system of equations is similar to that obtained within the (2+1)-dimensional scalar electrodynamics (the Abelian-Higgs model). In our case, however, we have the high-derivative terms (strongly nonlinear) instead of nonlinear (ϕ^3) terms for the scalar field. A possible way to study field equations (19) and (20) is the linearization procedure developed in [2]. For our purposes, however, we need non-perturbative solutions which are important in a study of topologically interesting effects.

Let us choose the static vortex-like ansatz for (19) and (20). Namely, in cylindrical coordinates (r, θ) we set

$$\chi^1(X^A) = F(r)\cos\nu\theta \qquad \chi^2(X^A) = F(r)\sin\nu\theta \tag{21}$$

and

$$W_r(X^B) = 0 \qquad W_\theta(X^B) = W(r) \qquad W_3 = 0 \tag{22}$$

where $r^2 = X^A X_A$. Note that the substitution of (21) and (22) into (19) and (20) results in very tedious equations for $F(r)$ and $W(r)$; we omit them here. However, after the choice $W(r) = \nu/r$ these equations become essentially simpler. It is clear that with this choice $\boldsymbol{W}(\boldsymbol{r})$ turns out to be a pure gauge but it is a topologically non-trivial solution. In this case, the disclination current is found to be normal to the plane. The strength of this current is determined via the Frank vector $\boldsymbol{\omega}$ which has a form $\boldsymbol{\omega} = (0, 0, \omega)$ where rot $\boldsymbol{W} = \omega\delta^2(\boldsymbol{r})$. Note that in many aspects disclination vortices have a close analogy with the well known magnetic Abrikosov-Nielsen-Olesen vortices [20, 21].

One can find that both sides of (19) turn out to be zero for our choice of $W(r)$ whereas (20) may be reduced to the form

$$\frac{\mathrm{d}}{\mathrm{d}r}[Ag^3(r)-Bg(r)]=-\frac{1}{r}[Ag^3(r)-Bg(r)]. \tag{23}$$

After integration this equation is found to be

$$|Ag^3(r)-Bg(r)|=g_0/r \tag{24}$$

where $g(r)=\mathrm{d}F(r)/\mathrm{d}r$, $A=\lambda/2+\mu$, $B=\lambda+\mu$ and g_0 is an integration constant. It should be stressed that this equation is almost the same as in the three-dimensional case for the disclination monopole [18]. The dimensional scaling, however, takes place. Namely, the characteristic behaviour of all physical values for planar vortices becomes like $1/r$ instead of $1/r^2$ in three space dimensions for the disclination monopole. For example, components of the stress tensor σ_1 are found to be $\sigma_1^X=g_0\cos\nu\theta\cos\theta/r$ and $\sigma_1^Y=g_0\cos\nu\theta\sin\theta/r$. The solution of (24) is obtained in the following form:

$$g(t)=\begin{cases} g_1(t)=N_0\cosh[\frac{1}{3}\cosh^{-1}(1/t)] & t\leqslant 1 \\ g_2(t)=-N_0\cos[\frac{1}{3}\cos^{-1}(1/t)+\frac{1}{3}\pi] & t\geqslant 1 \end{cases} \tag{25}$$

where $N_0=2(B/3A)^{1/2}$, and the universal dimensionless parameter $t=r/r_0$ is introduced. By analogy with [18] the region of the core of disclination vortex is clearly established, $r_0=(27g_0^2A/4B^3)^{1/2}$ defines the core radius. The analysis of (25) shows that the function $F(r)$ tends to a constant, F, when $r\to\infty$, whereas the disclination fields W_A tend clearly to zero as $1/r$. Thus, the solutions (19) and (20) satisfy the Dirichlet data for χ^i and the homogeneous Neumann data for W_a. It should be noted that equation (24) contains three real roots in the region $t\geqslant 1$. One can easily check, however, that two other solutions of (24) tend to a non-zero constant when $t\to\infty$. In this case, $F(t)$ tends to infinity at $t\to\infty$. It is clear that these solutions must be omitted. It is interesting to note that the value of F is calculated directly from (24), i.e. only via the model parameters. In the Higgs models this parameter, which characterizes the spontaneous symmetry breaking in a system, was introduced as an additional parameter of the model. The characteristic behaviour of $g(r)$ and $F(r)$ is shown in figure 1.

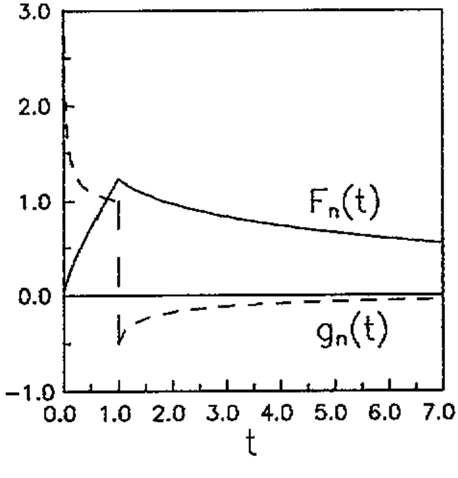

Figure 1. The functions $F_n(t)=F(t)/N_0r_0$ (solid line) and $g_n(t)=g(t)/N_0$ (dashed line) are presented. The point $t=1$ ($r=r_0$) corresponds to the disclination core radius.

4. Electronic properties

To investigate electronic properties of materials with continuously distributed dislocations and disclinations, we have proposed in [14] a self-consistent approach. Within this approach the electronic fields are introduced in a gauge invariant form. For this purpose, we have used the effective mass and deformation potential theories and constructed the continuum Lagrangian that is invariant under the inhomogeneous action of group G_3. It was shown that primary free electrons turn out to interact with disclination fields. The interaction of electrons with dislocations appears only via the deformation potential. In three space dimensions, we have found that this theory gives the known results for screw dislocations [22]. Let us consider here the two-dimensional problem in detail.

In the framework of the effective mass approximation the effective Lagrangian for the one-particle electron wavefunction $\psi(\boldsymbol{r})$ takes the following form:

$$L_\psi = \tfrac{1}{2}\{\mathrm{i}\hbar[\psi^+(\boldsymbol{r})D_3\psi(\boldsymbol{r}) - (D_3\psi^+(\boldsymbol{r}))\psi(\boldsymbol{r})] - (\hbar^2/m^*)[D_A\psi^+(\boldsymbol{r})D^A\psi(\boldsymbol{r})]\} \tag{26}$$

where $D_a\psi(\boldsymbol{r}) = (\partial_a - \mathrm{i}W_a)\psi(\boldsymbol{r})$ is the covariant divergence and m^* is an effective electron mass. Here we have used the complex representation for $\psi(\boldsymbol{r})$ (recall that $SO(2) \equiv U(1)$). The stationary Schrödinger equation in the effective mass approximation can be written as

$$-(\hbar^2/2m^*)(\partial_A - \mathrm{i}W_A)^2\psi_E(\boldsymbol{r}) = E\psi_E(\boldsymbol{r}). \tag{27}$$

Obviously, this equation should be added to a system of field equations for defects presented in section 2. It should be noted, however, that the self-consistent analysis of these equations is a very difficult problem. We will study (27) by considering disclination fields as external fields. Let us consider the disclination vortex (22). In this case we get the well known equation describing a charged point particle interacting with a point vortex [23]

$$-(\hbar^2/2m^*)(\nabla - \mathrm{i}\nu\nabla\theta)^2\psi_E(\boldsymbol{r}) = E\psi_E(\boldsymbol{r}). \tag{28}$$

Here we have used the fact that (20) can also be written as $\boldsymbol{W}(\boldsymbol{r}) = (\omega/2\pi)\nabla\theta$. As has been shown [23], the solution of (28) has to be taken in the form $\psi_E(\boldsymbol{r}) = e^{\mathrm{i}\nu\theta}\psi^0_k(r,\theta)$. In this case (28) is reduced to the free equation with the wavefunction obeying a non-trivial boundary condition, $\psi^0_k(r, 2\pi) = e^{-\mathrm{i}2\pi\nu}\psi^0_k(r, 0)$. Thus, a topological phase (a kind of Berry's phase) arises which depends on the disclination flux. As has been shown in [8], it results in the additional scattering of electrons thus changing the transport characteristics of disclinated materials. In particular, the relaxation time was found to be proportional to $\sin^{-2}\pi\nu$ in the presence of disclination vortices. Note that such an oscillating behaviour is peculiar to the AB effect.

It is known (see, e.g. [24]) that there are no restrictions on the value of ν apart from $\nu > -1$ for topological reasons. In real elastic systems, however, disclinations with small values of the Frank index ν ($\nu = \omega/2\pi$) are energetically preferable. It is of interest that in crystals the values of ν must conform to the point symmetry group of the underlying lattice. Namely, if we have the axis of m-fold symmetry, then the available values of ν are equal to $\nu = k/m$ where k is integer and fixes some restriction from above [24]. Thus, we conclude that the available values of ν in crystals are in fact 'quantized': $\nu = \frac{1}{6}, \frac{1}{4}, \frac{1}{3}, \ldots$. For integer values of ν the disclination vortex will have no influence on the topologically-induced resistivity. It is important to note that the results obtained in this section are formally equivalent to those for the electromagnetic AB effect. In fact, we have considered here only the topological part of the interaction. In the next section we will study the interaction of electrons with the lattice. As we shall see, this interaction is very important and can essentially affect the topological effect.

5. Deformation potential

In most of the real systems the interaction between electrons and acoustic waves should be taken into account. This problem can be studied in the framework of the deformation potential theory. In the isotropic case, the deformation potential is defined to be $W_d(X^C) = -(G/2)\,\mathrm{Sp}\,E_{AB}(X^C)$ where G is the interaction constant. Thus, the interaction Lagrangian takes the form

$$L_{\mathrm{int}} = -\psi^+(\boldsymbol{r})\,W_d(\boldsymbol{r})\psi(\boldsymbol{r}). \tag{29}$$

The interaction constant G can be estimated analogously to the three-dimensional case (see, e.g. [25]). Namely, the Fermi energy is determined in two space dimensions as $E_{\mathrm{F}} = 2\pi\hbar^2\rho/gm^*$, where ρ ($\rho = N/S$) is the (surface) density of conducting electrons and g denotes the degeneracy of electron levels. The change of the Fermi energy due to a deformation can be written in the form $\delta E_{\mathrm{F}} = -(\Delta S/S)E_{\mathrm{F}}$. On the other hand, the change of S can be expressed via the strain tensor: $\Delta S/S = \frac{1}{2}\,\mathrm{Sp}\,E_{AB}$. Supposing that the electron dispersion $E(\boldsymbol{k}, E_{AB}) = E(\boldsymbol{k}) - (G/2)\,\mathrm{Sp}\,E_{AB}$ keeps the same form up to the Fermi energy, one can estimate that $G = E_{\mathrm{F}}$. Taking into account the interaction term we rewrite the Schrödinger equation (28) in the form

$$\left(-\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}r\frac{\mathrm{d}}{\mathrm{d}r} + \frac{(j-\nu)^2}{r^2} - \frac{m^*G}{\hbar^2}\,\mathrm{Sp}\,E_{AB}\right)u_E^j(r) = k^2 u_E^j(r). \tag{30}$$

Here we have used the ansatz $\psi_E(\boldsymbol{r}) = \Sigma_j\, u_E^j(r)\,\mathrm{e}^{\mathrm{i}j\theta}/\sqrt{2\pi}$, $j = 0, \pm 1, \pm 2, \ldots$; and $k^2 = 2m^*E/\hbar^2$. For the vortex solution (19) and (20) we find that $\mathrm{Sp}\,E_{AB} = g^2(r) - 2$ where $g(r)$ is determined by (25). It is beyond the scope of our paper to study (30) in detail. We are interested here only in qualitative results. Let us rewrite the potential of (30) in the form (cf [14])

$$U = \begin{cases} E_{\mathrm{F}} - K_1 \cosh^2[\frac{1}{3}\cosh^{-1}(1/t)] + K_2/t^2 & t \leqslant 1 \\ E_{\mathrm{F}} - K_1 \cos^2[\frac{1}{3}\cos^{-1}(1/t) + \frac{1}{3}\pi] + K_2/t^2 & t \geqslant 1 \end{cases} \tag{31}$$

where $K_1 = DE_{\mathrm{F}}$ and $K_2 = K_2(j) = (j-\nu)^2\hbar^2/2m^*r_0^2$ are positive dimensional parameters, $D = 2B/3A = 4(\lambda+\mu)/3(\lambda+2\mu)$; clearly, $\frac{2}{3} \leqslant D \leqslant \frac{4}{3}$. The characteristic behaviour of (31) is as follows: for $t \to 0$ the third term in (31) tends to infinity as t^{-2} whereas the second term diverges as $-t^{-2/3}$. For $t \to 1^-$ one obtains $U \to E_{\mathrm{F}} + K_2 - K_1$; for $t \to 1^+$, $U \to E_{\mathrm{F}} + K_2 - K_1/4$. For $t \to \infty$ both terms in (31) tend to zero as t^{-2}. As a result, we have two possibilities for large t. Namely, for $t \to \infty$ $U(t)$ may tend to E_{F} both from above and from below. To analyse (31), it is convenient to use the substitution $\cosh 3\phi = 1/t$ for $t \leqslant 1$ and $\cos 3\psi = -1/t$ for $t \geqslant 1$, respectively (see [26]). In this case we get

$$U = U(\phi, \psi) = \begin{cases} E_{\mathrm{F}} - K_1 \cosh^2\phi + K_2 \cosh^2 3\phi & 0 \leqslant \phi \leqslant \infty \\ E_{\mathrm{F}} - K_1 \cos^2\psi + K_2 \cos^2 3\psi & \pi/3 \leqslant \psi \leqslant \pi/2. \end{cases} \tag{32}$$

Let us study the core region, $t \leq 1$. It is useful to introduce the parameter $\alpha = K_1/6K_2 = 2\pi\rho D r_0^2/3g(j-\nu)^2$. A simple analysis shows that in the core region the curve $U(t)$ will cross the axis $E = E_F$ when $\alpha \geq \frac{1}{6}$. For $\alpha \geq \frac{3}{2}$ we have found that $U(t)$ has a minimum at the point determined by the condition $\cosh^2 \theta_{min} = \frac{1}{2} + [(1+2\alpha)/16]^{1/2}$. The depth of the potential well is found to be

$$|U_0| = K_1[6\alpha - 1 + (1+2\alpha)^{3/2}]/12\alpha. \tag{33}$$

When α increases, the depth of the potential well rapidly increases as well and can reach the region $E < 0$ at $\alpha \geq \alpha_{cr}$ where $\alpha_{cr} \sim 3.2$ for $D = 1$.

Consider now the region $t \geq 1$. The analysis shows that for $\frac{2}{3} \leq \alpha \leq \frac{3}{2}$ the curve $U(t)$ crosses the axis $E = E_F$. At $\alpha < \frac{2}{3}$ the point $U(1^+)$ lies above this axis whereas at $\alpha > \frac{3}{2}$ it lies below and tends to E_F for $t \to \infty$ from below. The characteristic behaviour of the potential is shown in figure 2. One can see that for $E < E_F$ electron states are localized (basically in the core region). The scattered states ($E > E_F$) will acquire the additional (to the topological) phaseshift due to the deformation potential. In the general case, the value of this shift can be determined by a careful analysis of (30). In particular, when $\rho r_0^2 \ll 1$, the deformation potential is found to be small. In this case, the perturbation theory can be used, and an additional phaseshift is calculated to be small as well. Thus, even at the high energy ($E > E_F$) an electron can 'see' the lattice via the additional phaseshift. When the electron density is increased the additional phaseshift becomes considerable and can, in principle, compensate the topological phase. It will reflect in the resistivity properties of the material. The experimental verification of this prediction would be very interesting.

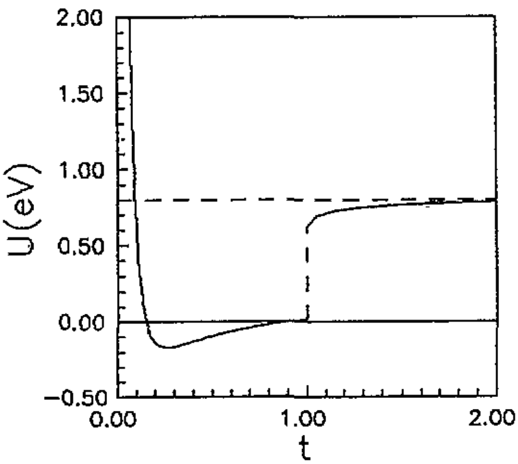

Figure 2. The effective potential (31) is shown. The parameter set is used: $D = 1$, $j = 0$, $\nu = 0.25$, $m^* = 0.5$ MeV, $r_0 = a_0$, $a_0 = 3.8$ Å, $\rho = 3.3 \times 10^{14}$ cm^{-2}, $g = 2$, so that $K_1 = E_F = 0.8$ eV and $K_2 = 0.0168$ eV.

In conclusion, in the framework of the EK gauge theory of topological defects rewritten for planar elastic systems we have established that rotational defects can affect the electronic properties of elastic materials. Namely, the wavefunction of a conducting electron is found to acquire the topological phase in the presence of the disclination vortex, which results in disclination-induced resistivity of materials with a specific oscillating behaviour. In our opinion, the experimental study of this effect in layered materials would be very important.

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